First-Principles Derivation of Causal Relativistic Dissipative Hydrodynamic Equation in Energy Frame with Stable Equilibrium State from Kinetic Equation by Renormalization-Group Method(Digest_要約)

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We obtain first-order relativistic dissipative hydrodynamics through solving the equation for the distribution function of the relativistic Boltzmann equation in the asymptotic regime as the invariant manifold of the dynamical system, on the basis of the renormalization-group (RG) method in a systematic manner with no ad-hoc assumption. In the present analysis based on the RG method, the perturbative expansion of the distribution function with respect to the spatial derivative with the Knudsen number, i.e., the ratio of the average particle distance over the mean free path, is first performed with the zeroth-order being the local equilibrium distribution function; the dissipative effect is taken into account as a deformation of the distribution function made by the spatial inhomogeneity as the perturbation. We show that the local equilibrium distribution function is identical to the Jüttner function expressed in terms of the five hydrodynamic modes, which are naturally identified with the same number of the zero modes of the linearized collision operator, i.e., the collision invariants. After defining the inner product in the function space spanned by the distribution function, the deviation from the Jüttner function that gives rise to the dissipative effects is constructed so that it is precisely orthogonal to the zero modes with respect to the inner product. Thus, any ansatz, such as the so-called conditions of fit used in the standard methods in an ad-hoc way, are not necessary, and rather the conditions of fit can be obtained as the orthogonality of the the deviation to the zero modes. In fact, the deviation turns out to satisfy the conditions of fit defining the energy frame, i.e., the Landau-Lifshitz one, provided that the macroscopic-frame vector, which defines the local rest frame of the flow velocity, is independent of the momenta of constituent particles, as it should. Our deviation thus suggests that the relativistic dissipative hydrodynamic equation must be defined in the energy frame, if it is consistent with the underlying kinetic equation. Furthermore, based on the positive definiteness of the inner product, we present an analytic proof that the first-order relativistic dissipative hydrodynamic equation in the energy frame has a stable equilibrium state. We also elucidate that the problematic Burnett term does not

affect the resultant equation owing to the nature of the hydrodynamic modes as the zero modes.

The second-order relativistic dissipative hydrodynamics is constructed as a mesoscopic dynamics of the relativistic Boltzmann equation. The mesoscopic dynamics occupies an intermediate level between the descriptions by hydrodynamics and kinetic theory. A basic observation presented in the extraction of the mesoscopic dynamics from the relativistic Boltzmann equation is to incorporate some excited (fast) modes of the linearized collision operator as additional components to the invariant manifold originally spanned by the zero modes for the first-order relativistic dissipative hydrodynamics. In fact, we develop the doublet scheme, i.e., a generic framework in the RG method to extract a mesoscopic dynamics from an evolution equation, on the basis of the following consistency condition and general principle of the reduction theory of the dynamics: (A) the resultant dynamics should be consistent with the slow dynamics obtained by employing only the zero modes in the asymptotic regimes; (B) the resultant dynamics should be as simple as possible because we are interested to reduce the dynamics to a simpler one. The mesoscopic dynamics of the relativistic Boltzmann equation obtained by the doublet scheme in the RG method has the same form as Israel-Stewart's fourteen-moment equation in the energy frame, i.e., a typical equation of the second-order relativistic dissipative hydrodynamics, but the microscopic formulae of the coefficients, e.g., the transport coefficients and relaxation times, are different. It is found that our theory leads to the same expressions for the transport coefficients as given by the Chapman-Enskog expansion method in contrast to Israel-Stewart's equations, and suggests novel formulae of the relaxation times expressed in terms of relaxation functions which allow a natural physical interpretation of the relaxation times. On the basis of the positive definiteness of the inner product as in the case of the first-order relativistic dissipative hydrodynamic equation, we show that the second-order relativistic dissipative hydrodynamic equation in the energy frame derived by the RG method has a stable equilibrium state, and respects causality, which means that the fluctuation around a static solution propagates with a finite speed that is slower than the speed of light. Furthermore, we demonstrate that the distribution function which is explicitly constructed in our theory provides a novel ansatz for the functional form of the distribution function that should be adopted in the fourteen-moment method proposed by Israel and Stewart.